V Semester B.A./B.Sc. Examination, November/December 2014
(Semester Scheme) (N.S.) (2013-14 and Onwards)
MATHEMATICS – V

Time : 3 Hours Max. Marks : 100

***Instructions** : Answer **all** questions.*

I. Answer **any fifteen** questions : **(15×2=30)**

1) In a vector space V(F) show that $(-a)\alpha = -(a\alpha), \forall a \in F, \alpha \in V$.

2) Show that $S = \{(x, y, z) / x = 2y = 3z\}$ is a subspace of $V_3(R)$.

3) Examine for linear dependence of the vectors (2, 1, 3), (–3, 4, 5), (1, –2, 1) in $V_3(R)$.

4) Show that $T : V_3(R) \to V_2(R)$ defined by T(x, y, z) = (x + z, y – z) is a linear transformation.

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5) Find the matrix of the linear transformation $T : V_3(R) \to V_2(R)$ defined by T(x, y, z) = (2x + 3y, y + 3z) relative to standard bases.

6) Find the null space of the linear transformation $T : V_2(R) \to V_2(R)$ defined by T(x, y) = (x + y, x).

7) If the vector function $\vec{f}(t)$ has constant magnitude, show that $\vec{f}$ is perpendicular to $\frac{d\vec{f}}{dt}$ (provided $\frac{d\vec{f}}{dt} \neq 0$).

8) For a space curve define (i) Curvature, (ii) Torsion at any point.

9) Show that the necessary and sufficient condition for a curve in space to be a straight line is that the curvature K = 0 at all points.

10) Find the unit tangent vector at t = 2 for the curve $\vec{r} = (t^2 + 1)\hat{i} + 2t\hat{j} - (t^3 + t)\hat{k}$.

11) Find the equation of the tangent plane to the cylinder $x^2 + y^2 = 4$ at $(1, \sqrt{3}, 2)$.

12) Find the spherical co-ordinates of the point whose Cartesian co-ordinates are $\left(\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}, \sqrt{2}\right)$.

13) Find the maximal directional derivative of $x^2 + yz^2$ at (1, –1, 3).

14) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$ find $\operatorname{div}(r^n \vec{r})$ in terms of r.

15) Show that the vector field $\vec{F} = (6xy + z^3)\,\hat{i} + (3x^2 - z)\,\hat{j} + (3xz^2 - y)\,\hat{k}$ is irrotational.

16) Find $\nabla^2\phi$ where $\phi = xz + zy + xy$.

17) If $F[f(x)] = \hat{f}(\alpha)$, prove that $F[f(ax)] = \frac{1}{a}\hat{f}\left(\frac{\alpha}{a}\right), (a > 0)$.

18) Find the inverse Fourier transform of $\hat{f}(\alpha) = \begin{cases} 1 & \text{for } |\alpha| \le a \\ 0 & \text{for } |\alpha| > a \end{cases}$.

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19) Find the Fourier cosine transform of $f(x) = e^{-5x}$.

20) Prove that $F_s[f'(x)] = -\alpha F_c[f(x)]$.

II. Answer **any four** of the following :

1) Prove that the set $V = \{a + b\sqrt{2} \,/\, a, b \in Q\}$ where Q is the field of ratio numbers form a vector space with respect to the addition and multiplica of rational numbers.

2) Show that the vectors (1, 2, –3), (1, –3, 2) and (2, –1, 5) span the vec (1, –2, 7) in $V_3(R)$.

3) Prove that every linearly independent subset of a finitely generated vec space V(F) form a part of a basis of V(F).

4) Given the matrix $A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 3 & 0 \end{bmatrix}$ determine the linear transforma $T : V_3(R) \to V_2(R)$ associated with A relative to bases B_1 = {(1, 0, 0), (0, 1, 0), (0, 0, 1)} and B_2 = {(3, 2), (1, 4)}.

5) State and prove Rank-Nullity theorem.

6) Show that $T : V_3(R) \to V_3(R)$ defined by $T(x, y, z) = (2x, 3z, y)$ is non-sing and find its inverse.

III. Ans er **any four** questions : (4×5=20)

1) erive the expressions for curvature and Torsion in terms of the derivatives $\vec{r}$ for a space curve with respect to the arc length s.

2) or the space curve $x = t$, $y = t^2$, $z = \frac{2}{3}t^3$, find K and τ.

3) ind the angle between the unit tangent vectors drawn to the curve $= a \cos 2t$, $y = a \sin 2t$, $x = at$, at the points $t = \frac{\pi}{6}$ and $t = \frac{\pi}{4}$.

4) Find the equations of the tangent plane and normal line to the surface $xy + yz + zx = -1$ at $(-2, 3, 5)$.

5) Find the angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at $(1, -2, 1)$.

6) Express the vector $\vec{f} = z^2\hat{i} - zx^2\hat{j} + 4y\hat{k}$ in cylindrical co-ordinates.

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IV. A wer **any three** of the following : (3×5=15)

If $\vec{f} = 2x\hat{i} + 3y\hat{j} + 4z\hat{k}$ and $\phi = xy^2z^3$, find (i) $\vec{f}.\nabla\phi$ (ii) $\nabla|\vec{f}|^2$.

Find the directional derivative of $\phi(x, y, z) = xyz - xy^2z^3$ at $(1, 2, -1)$ in the direction of the vector $\hat{i} - \hat{j} - 3\hat{k}$.

Prove that $\nabla \times (\phi\vec{f}) = \phi(\nabla \times \vec{f}) + \nabla\phi \times \vec{f}$ for any scalar function ϕ and vector function $\vec{f}$.

) Find $\nabla \times (\nabla \times \vec{f})$ given $\vec{f} = x^2y\hat{i} + y^2z\hat{j} + z^2x\hat{k}$.

) Show that $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$ where $r^2 = x^2 + y^2 + z^2$.

V. Answer **any three** of the following : (3×5=15)

1) Express $f(x)=\begin{cases}1 & \text{for } |x|\le 1\\ 0 & \text{for } |x|>1\end{cases}$ as a Fourier integral. Hence evaluate $\int_0^\infty \frac{\sin x}{x}\,dx$.

2) Find the Fourier transform of $f(x)=\begin{cases}1-|x| & \text{for } |x|\le 1\\ 0 & \text{for } |x|>1\end{cases}$.

3) Show that $xe^{-\frac{x^2}{2}}$ is self reciprocal under the Fourier Sine Transform.

4) Find the Fourier Cosine Transform of $f(x)=\frac{1}{1+x^2}$.

5) Given $F_S[xe^{-ax}]=\sqrt{\frac{2}{\pi}}\times\frac{2a\alpha}{(a^2+\alpha^2)^2}$ and $F_C[e^{-ax}]=\sqrt{\frac{2}{\pi}}\frac{a}{a^2+\alpha^2}$, find $F_C[xe^{-ax}]$.

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